# ITI9200 – Category theory 2024

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28/01/2024

# What is this 'Category Theory' about

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- all objects of a given type can be collected in a **class**
- such objects form coherent conglomerates, allowing for **relations** between structures to form
- far from being rare, these relational conglomerates are **pretty common** and arise at every corner.

«Category theory can be seen as a theory of *systems* and *processes*.» (Hu-Vicary, 2021)

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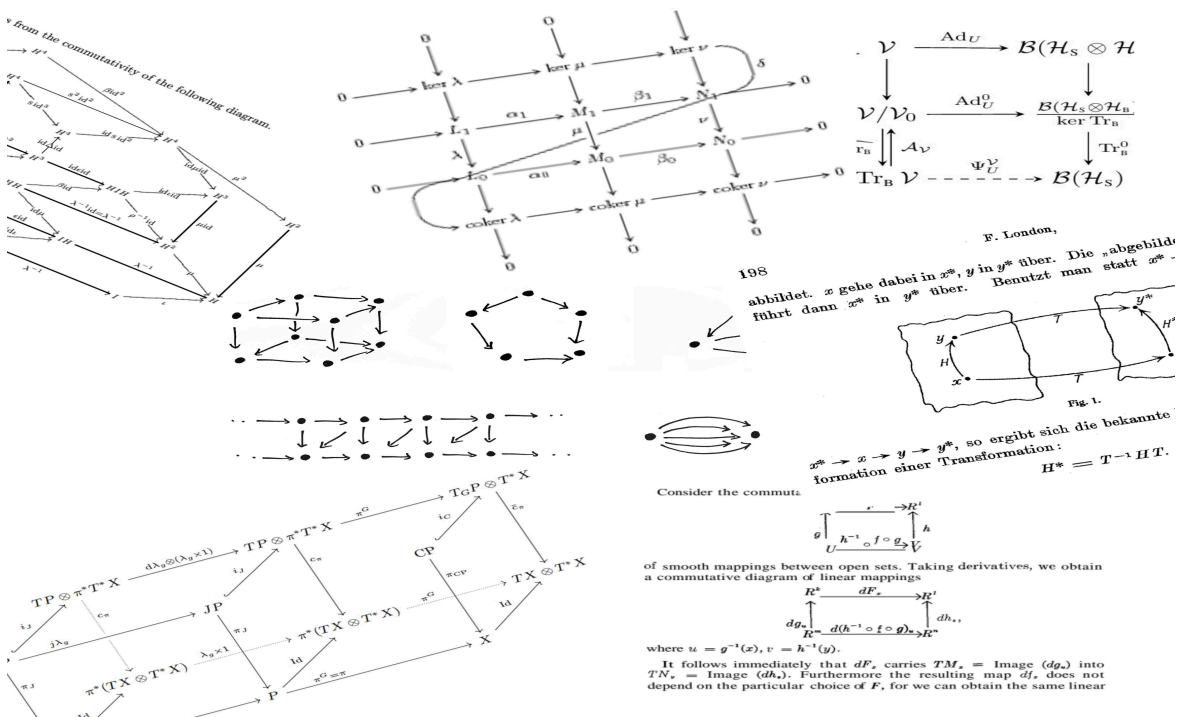
Information about a problem is presented as a diagram (an oriented graph of a very special kind):



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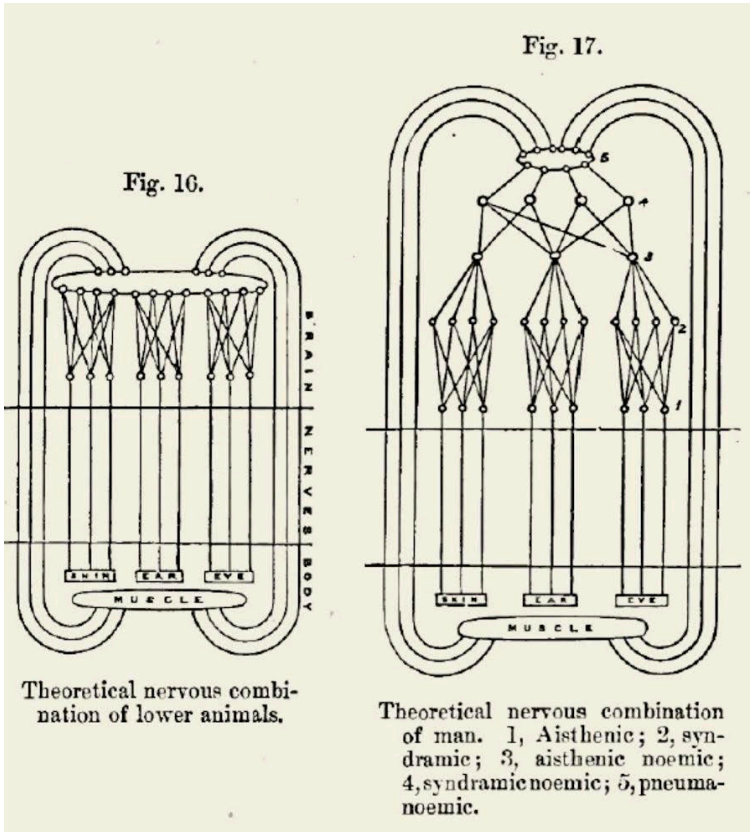
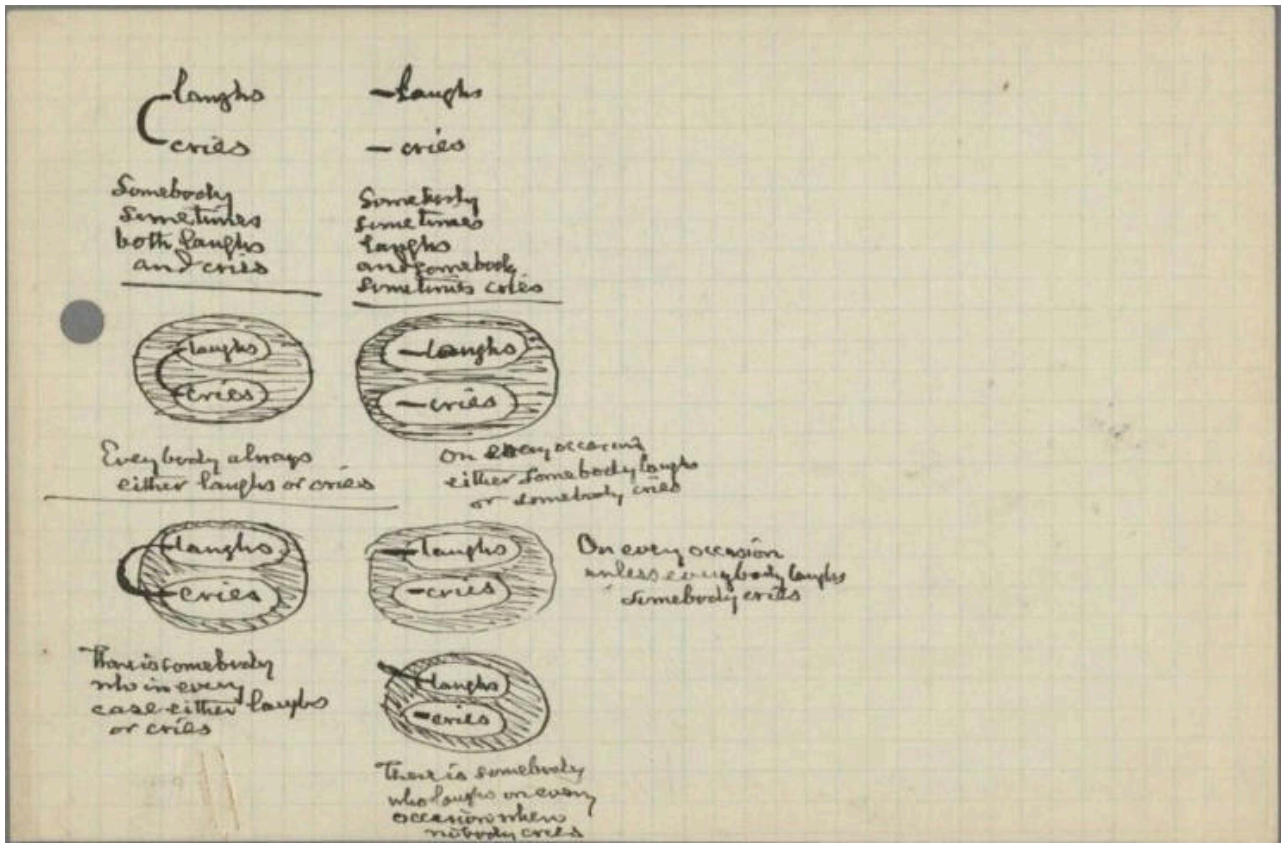
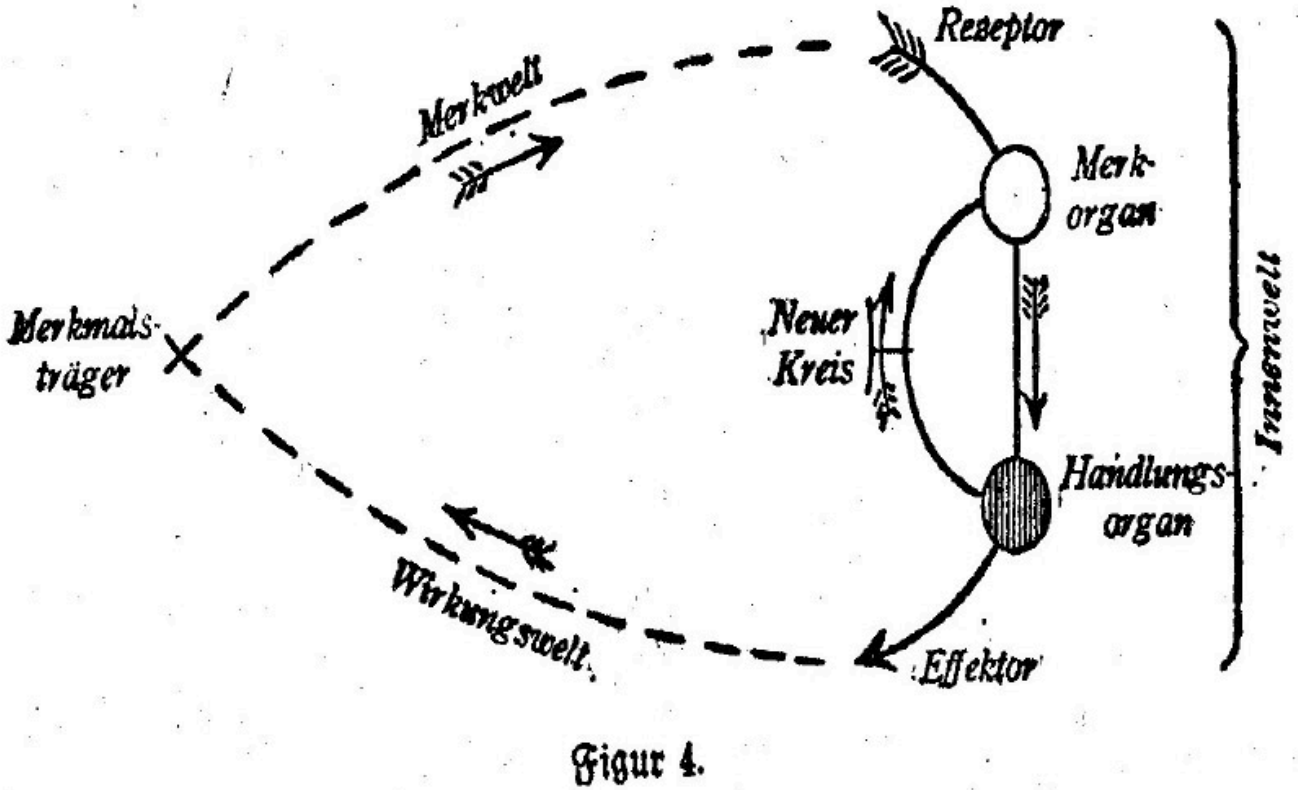
All the following things relate to Category Theory or to the particular philosophy that generated it. (The choice to present them in chronological disorder is deliberate.)




Alfred Smee. *Instinct and Reason Deduced from Electrobiology*, 1850.

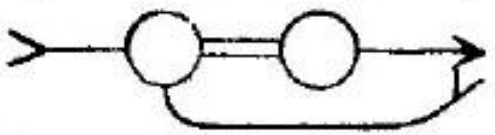
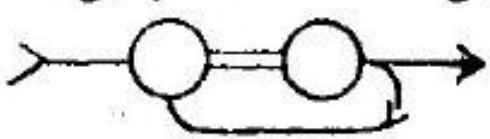



C.S. Peirce, *Prolegomena to an Apology of Pragmaticism*, 1906



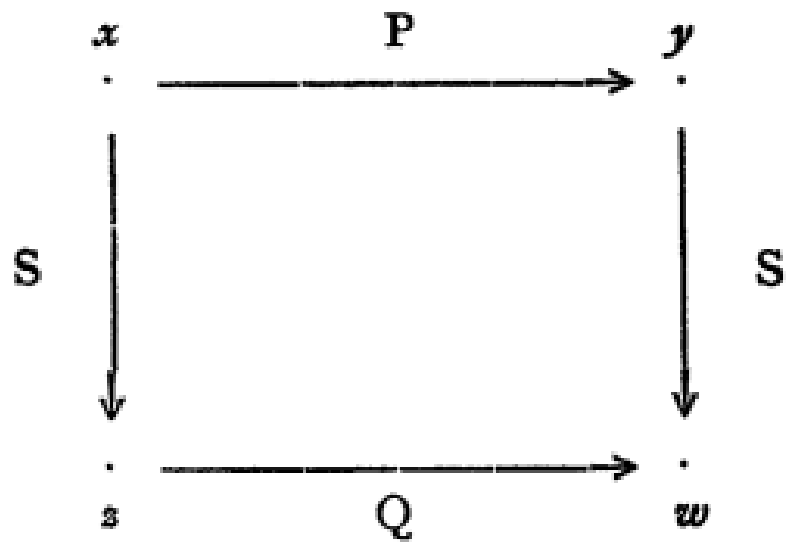

Figur 4.

J. J. von Uexküll. “Early Scheme for a circular Feedback Circle” from *Theoretische Biologie* 1920.

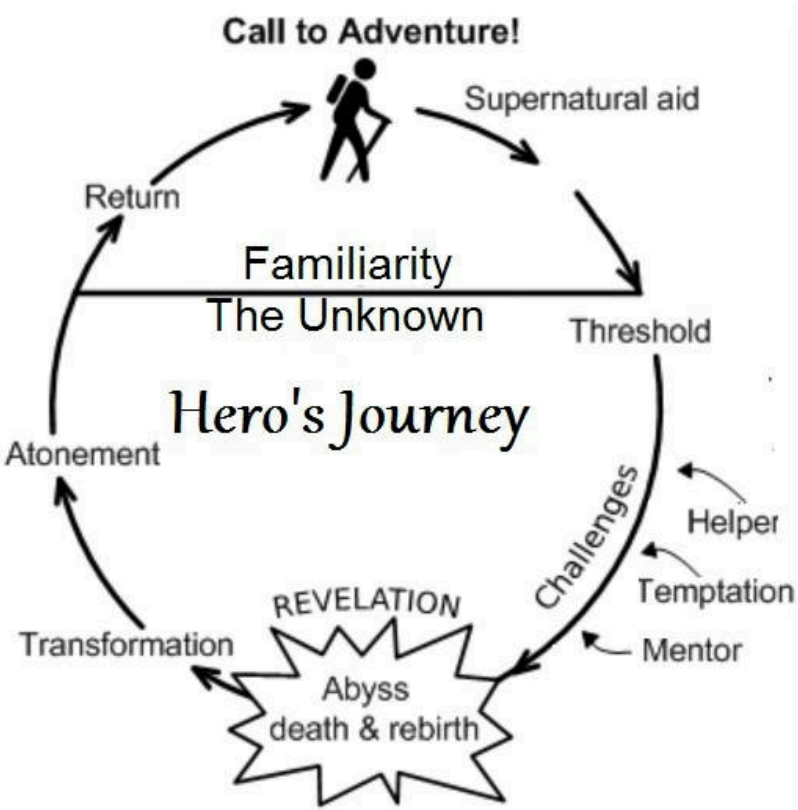
ſind zwei Fälle zu unterſcheiden: entweder wirk
Effektorenmuskeln durch beſondere ſenſible Nerve
beifolgende Schema zeigt.  Oder es
toriſchen Nerven übertragene Erregung durch
Rezeptoren zum Teil aufgefangen und dem N
 Dieſe Rezeptoren bilden das zentra
Helmholtz, das anatomiſch noch völlig im Dunkeln

J. J. von Uexküll. “Zirkuläre Schemen” from *Theoretische Biologie* 1920. Diagrammatic description of double feedback system of autonomic nerves in the brain.

to another, the correlate of the one has the relation Q to the correlate of the other, and *vice versa*. A figure will make this clearer. Let $x$ and $y$ be two terms having the relation P. Then there are to be two terms $z$, $w$, such that $x$ has the relation S to $z$, $y$ has the relation S to $w$, and $z$ has the relation Q to $w$. If this happens with every pair of terms such as $x$




B. Russell, *Introduction to Mathematical Philosophy*, 1919 (p 54).

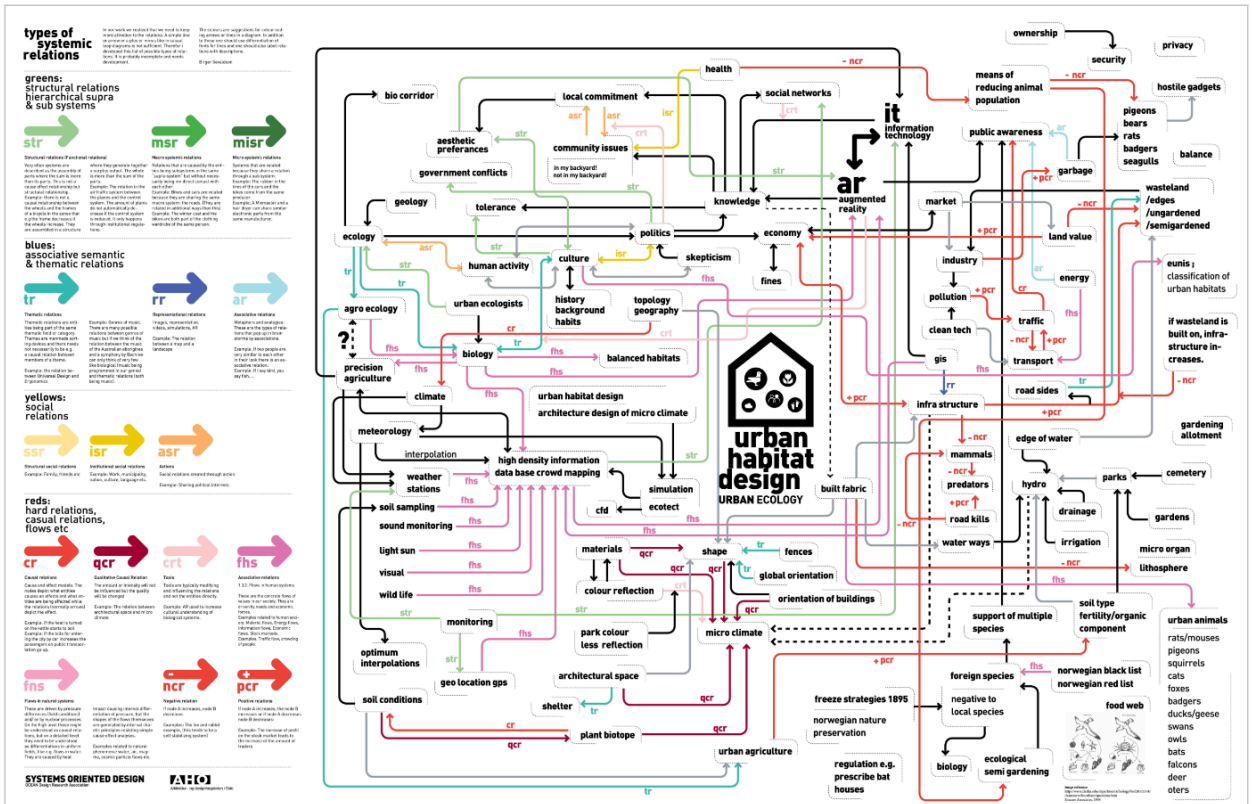



J. J. Campbell, *The Hero with a Thousand Faces*, 1949. (mythogenesis of heros' tales and folk stories)

There are some useful rules of thumb for how to gigamap. These rules have emerged through years of experience producing such maps and instructing students and professionals in gigamapping.

> Within the final and true world image everything is related to everything, and nothing can be discarded a priori as being unimportant. – Fritz Zwingy 1969

The concept of *gigamapping*, in System Oriented Design https://systemsorienteddesign.net




*Relational gigamapping*, from https://systemsorienteddesign.net/gigamap-relational/

«When this (एतद्, *etad*) exists, that comes to be. With the arising (उप्पाड, *uppada*) of this, that arises. When this does not exist, that does not come to be. With the cessation (निरोध, *nirodha*) of this, that ceases.»

—Samyutta Nikaya 12.61.

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The Buddhist doctrine of *pratītyasamutpāda* (skrt: प्रतीत्यसमुत्पाद, roughly: *co-dependent origination*) states that all phenomena (dharma) arise in dependence upon other phenomena.

In each atom the Buddhas of all times
Appear, according to inclinations;
While their essential nature neither comes nor goes,
By their vow power they pervade the worlds.

—Buddhāvataṃsaka Sūtra, 7:I, Bk 4

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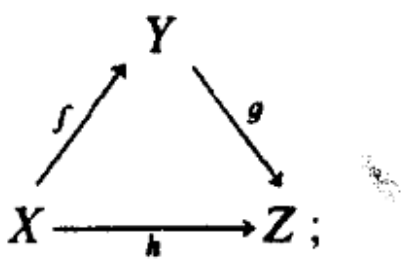
In Vedic mythology, when Indra dreams the world, he builds it as a spiderweb or network, with each crossing adorned with a jewel. Every *dharma* is a node in this network, and the surface of each jewel reflects every other, so that every thing that exists implies all the others.

Category theory starts with the observation that many properties of mathematical systems can be unified and simplified by a presentation with diagrams of arrows. Each arrow $f : X \to Y$ represents a function; that is, a set $X$, a set $Y$, and a rule $x \mapsto fx$ which assigns to each element $x \in X$ an element $fx \in Y$; whenever possible we write $fx$ and not $f(x)$, omitting unnecessary parentheses. A typical diagram of sets and functions is




S. Mac Lane, *Categories for the working Mathematician.* (1971)

A snippet of `haskell` code

```haskell
compose n = do
  n1 <- f n
  n2 <- g n1
  n3 <- h n2
  return n3
```

composing two partial functions.

# What is this course about

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- `fosco.loregian@gmail.com`
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Plus TAs or other teachers:

- N. Arkor (`https://arkor.co`)
- A. Laretto (`https://iwilare.com`)
- M. De Pascalis (`michele.de@taltech.ee`)

On the exam methods:

- Solve some exercises (approx assigned at **1/3** and **2/3** of the course);

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The course webpage: **http://tinyurl.com/ct-taltech-24**

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So tell us:

# *Who are you?*

Where are you from? Why would you like to learn CT? Do you know how to program? In what language(s)?