GOC

Categorical tools I

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This is the (co)end, my only (co)friend

Fosco Loregian (SISSA). Introduced by Yoneda and Kelly in the 60s, the formalism of (co)ends subsumes in a nifty way several well-known constructions

- in elementary **Category Theory** (natural transformations between two functors, Kan extensions, "tensor product" of functors, weighted limits),
- in **Abstract Algebra** (tensor product of R-modules, induced and coinduced representation of a group along a morphism),
- in **Geometry/Topology** (geometric realization of a simplicial set, the nerve-realization paradigm, the classifying space of a topological monoid),
- and in **less elementary Category Theory** (the theory of Benabou's profunctors) and less elementary geometry/topology (characterization and generalization of May's operads, Borsuk-Cordier-Porter's shape theory via profunctors, $\mathcal{V}$-weighted limits).

I will try convey the idea that an "endy" approach to Category Theory, Algebra and Geometry can turn involved arguments in neat proofs which "go by nonsense" until the end (no pun intended...).

The talk is adressed to people familiar with the categorical notion of limit, which is sufficient to follow the entire discussion: every notion that goes beyond the basics will be introduced and motivated (in particular, I assume no prior acquaintance with the various algebraic and geometric constructions in study).

References.

— S. Mac Lane, *Categories for the Working Mathematician*.
— S. Mac Lane, *The Milgram bar construction as a tensor product of functors*.
— E. Riehl, *Categorical Homotopy Theory*.
— J. Bénabou, *Distributors at work*.
— J. M. Cordier, T. Porter, *Shape Theory: Categorical methods of approximation*.
— S. Willerton, *Ends*, post @n-café.