Fibrational linguistics

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Caveat

This talk is two talks;

- one is for mathematicians, like me, and it's full of weird symbols and definitions;
- the other is way more informal and ends up explaining the 'spirit' of the project (or rather, my[1] stance towards philosophy of language);

[1] Opinions are my own, etc etc.

Intro

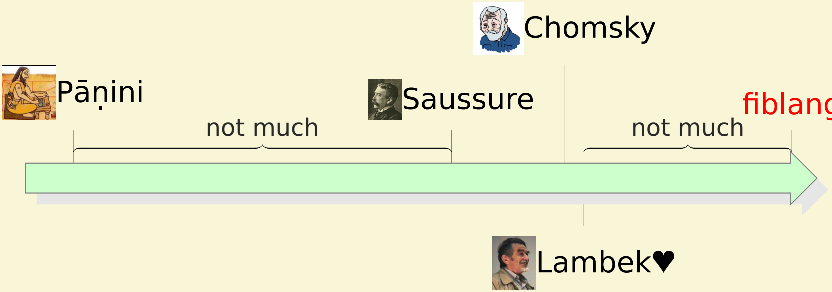
Since I was very young I loved language. I was amazed by the fact that there is more than one: why so many? Can I invent one myself (raise your hand if you didn't)? How do they function? Shouldn't we just stick to a single one and avoid ambiguity? What about words that can't be translated? What is the most expressive language around? Which one was the first, and how did people come up with that? . . . humans have coherent ways to convey meaning? What sorcery is that?

. . . I wasn't an easy child to raise.

This work is an attempt to appease my inner child.

Linguistics,
as a category thst

A brief, uninformed timeline of linguistics



A brief, uninformed timeline of linguistics

Pāṇini defines Sanskrit morphology in a system of 4k commented context-sensitive rewrite rules; all before India even had a writing system; badass move.

Saussure (1857-1913) cleans the slate of ill-founded assumptions on the shape of PIE, comes up w/ laryngeal theory; when he's 21 yo; badass move.

Chomsky (1928–) says hey, I heard you like languages, what about putting logic in the study of them?

This has made a lot of people very angry and been widely regarded as a bad move.

Lambek (1922-2014)[2]: hey guys,

- (the syntactic type system of) a [simplified version of a natural] language is a monoidal category!
- so we can transport 'facts' about those specific monoidal categories (called 'pregroups') and encode well-formedness of sentences in a language;
- all this, to the effect that when you 'speak' a language you do something like natural deduction in the category associated to the language.

[2]Building on prior work of Adjukiewicz and Bar-Hillel. There's also a beautiful survey by Haskell Curry...

More formally, Lambek defined a sequent calculus:

a context containing a sub-context A

$$\frac{}{A \Rightarrow A}\, id \qquad \frac{\Gamma \Rightarrow A \quad \boxed{\Delta(A)} \Rightarrow B}{\Delta(\Gamma) \Rightarrow B}\, Cut$$

$$\frac{\Gamma \Rightarrow A \quad \Delta(C) \Rightarrow D}{\Delta(\Gamma, A\backslash C) \Rightarrow D}\, \backslash L \qquad \frac{A, \Gamma \Rightarrow C}{\Gamma \Rightarrow \boxed{A\backslash C}}\, \backslash R$$

$A\backslash C \bullet C = A$

$$\frac{\Gamma \Rightarrow B \quad \Delta(C) \Rightarrow D}{\Delta(C/B, \Gamma) \Rightarrow D}\, /L \qquad \frac{\Gamma, B \Rightarrow C}{\Gamma \Rightarrow \boxed{C/B}}\, /R$$

$C \bullet C/B = B$

$$\frac{\Delta(A, B) \Rightarrow D}{\Delta(A \bullet B) \Rightarrow D}\, \bullet L \qquad \frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A \bullet B}\, \bullet R$$

FIGURE 2.1. The Lambek sequent calculus

This means (well, sort of) that the phenomenon happening right now in this very moment is possible because we have category theory.

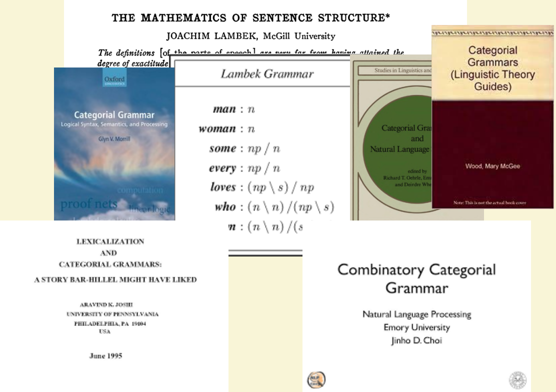
This means that resistance (to adopt structural thinking) is futile.

Categories are not a language; they are languages.

Few revelations had a comparable influence on my mathematical beliefs.

I had to do something with this idea.

With the passing of years, I discovered that people actually used Lambek's idea a lot:



. . . and many others.

This is all great.

But something was missing: to me, categories are living beings, not mere syntactic objects. And language changes with use, communication can be obtained by trial-and-error, interpretations can vary (and be context dependent. . .).

So:

Question

Is there a way to employ category theory to describe the dynamics of language, besides its internal syntactic structure?

A speech-circuit

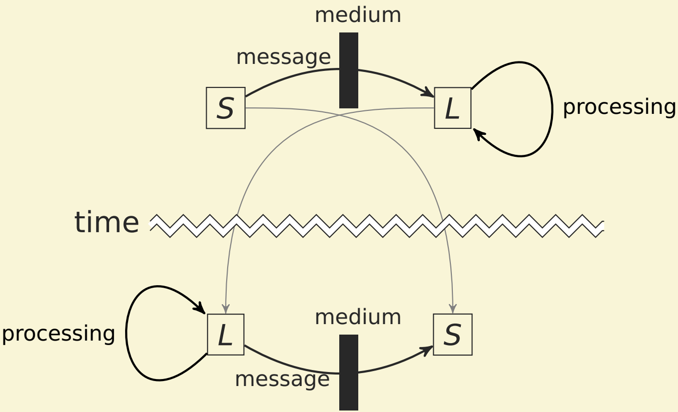
Saussure cited the canonical use of language as the cycle of a speechcircuit.

A speaker expresses a psychological idea by means of a physiological articulation.

The signal is transmitted through the medium by a physical process incident on a hearer who, from the consequent physiological impression, recovers the psychological idea.

The hearer then replies, becoming the speaker, and so the roles of speaker and hearer keep swapping, and the circuit cycles.

A speech-circuit



Can we use category theory to describe this process?

Fibrational Linguistics in a nutshell

This was the main motivation to embark on what became FibLang: try to describe the linguistics linguists do, with its questions and subtleties, using Mathematics.

> FibLang seeks a foundation for the process of interaction and collective construction of a shared deductive system, in which "speakers evaluate terms" and exchange the results of their computation for extending the expressiveness of $\mathcal{L}$.

Fibrational Linguistics in a nutshell

Typical questions that animate our attempt:

- How is the morphological complexity of a language linked to some invariant of the category that $\mathcal{L}$ presents?
- How to model the fact that language modification is a collaborative process, that modifies syntax in order to attain a goal (mutual understanding, avoid ambiguity,...)?
- Can we model linguistic pressure, linguistic diversity, mutual influences...in a mathematically precise way?

Fibrational Linguistics in a nutshell

Let's start simple and let's assume nothing apart from the fact that

- language is a category $\mathcal{L}$;
- what we do with language -filtering perceptions- is also a category, $\mathcal{E}$.

We experience* the world*, understanding* (parts of) it, and communicate representations† to others.

*whatever that means; explaining this pertains to philosophers of language.

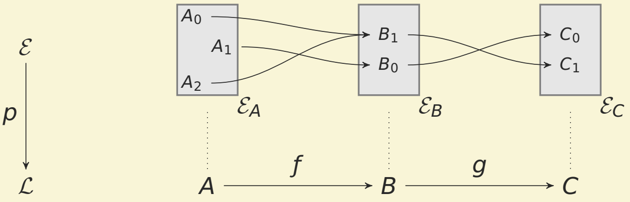
†this is what we want to describe.

Fibrational Linguistics in a nutshell

Of course, there must be some relation between $\mathcal{E}$ and $\mathcal{L}$: the mental image that we create of a sensible object is certainly related to the object itself in some way.

Intuitively, the logical relation in which perceived things stand shall be preserved in the mental image.

The notion of fibration is just the thing!



Put the *Fib* in FibLang

Definition

A functor $p^\sharp : \mathcal{E} \to \mathcal{L}$ is a discrete opfibration if, for every object E in $\mathcal{E}$ and every morphism $f : pE \to C$ in $\mathcal{L}$, there exists a unique morphism $h : E \to E'$ such that $p^\sharp h = f$.

From this it follows that each fiber $p^{-1}C$ is a discrete category (=a set). The uniqueness request now says that ‘fibers vary coherently’ according to the structure of $\mathcal{L}$.

Put the *Fib* in FibLang

Goal: read a textbook on fibrations and re-interpret the structure of the fibres of $p^\sharp$ over objects L as 'what p thinks L means', i.e. the totality of interpretations that p can give to a 'word' L.

Assumptions

A language is a category $\mathcal{L}$; a speaker of $\mathcal{L}$ is another category, fibered over $\mathcal{L}$; in this way, the totality of speakers of $\mathcal{L}$ is a (2-)category $Fib_{\mathcal{L}}$, and the totality of all speakers of all languages is a (2-)category Fib.

Theorems about fibrations can be interpreted as 'theorems' about the structure of language.

Motto: FibLang is the category theory of the categories $Fib_{\mathcal{L}}$ and Fib.

A parallel w/ categorical logic

In categorical logic, once a signature Σ is specified, we can build contexts as strings of declarations $\Gamma = (x_1 : \sigma_1, \dots, x_n : \sigma_n)$, and assess judgments like

$$\Gamma \vdash X : \tau$$

to express that in context Γ, a term X has type $\tau \in \Sigma$.

To every signature it can be associated a classifying category $\mathcal{C}(\Sigma)$, whose objects are contexts Γ above, and whose morphisms are suitable substitutions of terms one inside the other.

A parallel w/ categorical logic

Now, there is an equivalence between

- models for the theory that the signature prescribes;
- functors $\mathcal{C}(\Sigma) \to \mathbf{Set}$ that preserve finite products;
- certain fibrations $\left[p^\sharp \begin{array}{c} \mathcal{E} \\ \downarrow \\ \mathcal{C}(\Sigma) \end{array} \right]$ over $\mathcal{C}(\Sigma)$.

A parallel w/ categorical logic

Such a fibration has as fibre over a given $\Gamma \in \mathcal{C}(\Sigma)$ precisely the category/poset of judgments $X : \tau$ that are valid in context Γ.

A reasonable parallel with our model for language representation is that

- as much as a fibre of $\left[p^\sharp \begin{array}{c} \mathcal{E} \\ \downarrow \\ \mathcal{C}(\Sigma) \end{array} \right]$ over Γ is the set of 'judgments that can be deemed true' in context Γ,
- a fibre of $\left[p^\sharp \begin{array}{c} \mathcal{E} \\ \downarrow \\ \mathcal{L} \end{array} \right]$ over L is the set of meanings that can be attributed to L by $p^\sharp$.

A categorical toolkit

Tools: The Grothendieck construction

The Grothendieck construction asserts an equivalence between

- discrete opfibrations over $\mathcal{L}$;
- functors $\mathcal{L} \to \mathbf{Set}$.

$$\nabla : DFib/\mathcal{L} \rightleftarrows [\mathcal{L}, \mathbf{Set}] : \int$$

The equivalence identifies the fiber $p^{\leftarrow}L$ of a fibration over L and the set $F_p(L)$, canonically.

Tools: The Grothendieck construction

The total space of a fibration contains exactly the information needed to think of $\mathcal{L}$ as a 'theory' and as p as a 'model' of that theory, in a way that L

- has a fiber made up from what p thinks L means ('the set of meanings of L') mindful of the relations existing between objects in $\mathcal{L}$;
- is sent to a set $F_p L$ by the associated functor, mindful of the relations between objects in $\mathcal{L}$.

In both cases, the 'relations' are of course morphisms of $\mathcal{L}$.

Tools: the comprehensive factorization

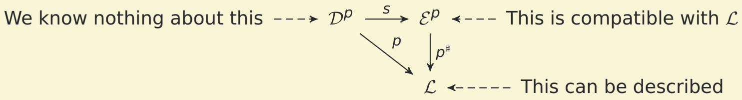
Street factorization

(or ‘comprehensive’ factorization of a functor) Let $F : \mathcal{C} \to \mathcal{D}$ be a functor; then we can factor F as

$$\mathcal{C} \xrightarrow{s} \mathcal{E} \xrightarrow{p^{\#}} \mathcal{D}$$

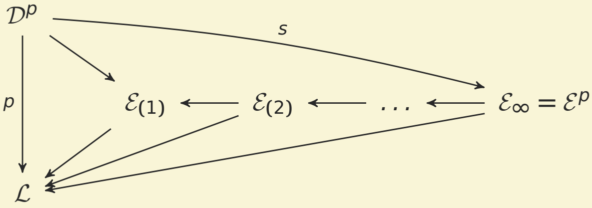
wher $p^{\#}$ is a fibration.

The functor s is called ‘initial’; we are not interested in what is an initial functor but they are pretty important in CT.



Definition

Given a speaker $\mathcal{D}^p \xrightarrow{p} \mathcal{L}$ factorizing as $\mathcal{D}^p \xrightarrow{s} \mathcal{E}^p \xrightarrow{p^\sharp} \mathcal{L}$, we call $p^\sharp$ the language framework of the speaker p.



Let’s do a tiny fragment of linguistics

Vocabulary acquisition

We can explore two models for vocabulary acquisition.

- by example: ‘Look, a cat!’;
- by paraphrasis/definition: ‘A cat is a tiny evil feline, possibly black’.

Cats, by example

Alice: Look, a cat!

Bob: A what? [*Alice points to a cat*]

Alice: That, a cat!

Bob: Oh!

What happened in that ‘Oh!’ can be mathematically modelled as a colimit in the language frameworks of Alice and Bob.

Cats, by example

Consider two fibrations $\left[\begin{smallmatrix} & \mathcal{E}^p \\ p^\sharp & \downarrow \\ & \mathcal{L}\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} & \mathcal{E}^q \\ q^\sharp & \downarrow \\ & \mathcal{L}\end{smallmatrix}\right]$, which we will call teacher and learner, respectively.

Suppose that, for some $L \in \mathcal{L}$ –the linguistic element to learn– we have that $\mathcal{E}^p_L \neq \varnothing$ and $\mathcal{E}^q_L = \varnothing$.

Fix a subset $S \subseteq \mathcal{E}^p_L$, called an example for L.

Then we can define a new category $\mathcal{F}^q$ forcefully stitching the teacher's idea into q's *tabula rasa*.

Cats, by example

Define a functor $T : \mathcal{F}^q \to \mathcal{L}$ agreeing with $\left[\begin{smallmatrix} \mathcal{E}^q \\ q^\sharp \downarrow \\ \mathcal{L} \end{smallmatrix} \right]$ on every fibre $L' \neq L$, and sending every object of S to L.

The new linguistic framework of q after learning L is the comprehensive factorization

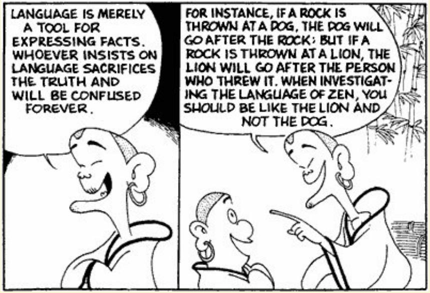
$$T = \left(\mathcal{F}^q \xrightarrow{s} \mathcal{E}^{\tilde{q}} \xrightarrow{\tilde{q}^\sharp} \mathcal{L} \right).$$

Idea: q ‘merges’ $\mathcal{F}^q$ with their mental image to turn the new setting (the one where q knows what a cat is) into another fibration.

Zen, by example

[. . .D]uring the *Flower Sermon*, Gautama did nothing but hold up a lotus flower, in silence. Upon seeing it, his disciple Mahākāśyapa was immediately enlightened.

Why did the simple display of a flower have such a profound effect upon Mahākāśyapa, and what can we learn from this lesson?



Explanation

A speaker p and a speaker q meet to discuss the theories of 18th century's philosopher Immanuel Kant.

What happened to the fibrations $p^\sharp, q^\sharp$ after they discussed?*

Man shot in Russia in argument over Kant

(Reuters) - An argument over the theories of 18th century philosopher Immanuel Kant ended in a man being shot in a grocery store in southern Russia.

RIA news agency quoted police in the city of Rostov-on-Don as saying a fight broke out between two men as they argued over Kant, the German author of "Critique of Pure Reason", without giving details of their debate.

* provided things do not go horribly wrong?

Explanation

Definition

Consider a speaker $p : \mathcal{D}^p \to \mathcal{L}$ and their language framework, i.e. the associated fibration $\left[\begin{array}{c} \mathcal{E}^p \\ p^\sharp \downarrow \\ \mathcal{L} \end{array} \right]$.

Fix moreover an object L of $\mathcal{L}$. An explanation for L according p is a finite diagram $D_L : \mathcal{A} \to \mathcal{L}$ such that the limit $L^\star$ of the diagram

$$\mathcal{A} \xrightarrow{D_L} \mathcal{L} \xrightarrow{\nabla p^\sharp} \mathbf{Set}$$

is a subset of the fibre $\mathcal{E}^p_L$.

If $L^\star = \mathcal{E}^p_L$, we call the explanation exact.

Cats, by definition

Consider two speakers $p : \mathcal{D}^p \to \mathcal{L}$, $q : \mathcal{D}^q \to \mathcal{L}$ and their respective language frameworks, i.e. the fibrations $\left[\begin{array}{c} \mathcal{E}^p \\ p^\sharp \downarrow \\ \mathcal{L} \end{array} \right]$ and $\left[\begin{array}{c} \mathcal{E}^q \\ q^\sharp \downarrow \\ \mathcal{L} \end{array} \right]$, which we will call *teacher* and *learner*, respectively.

Suppose that, for some $L \in \mathcal{L}$ –the linguistic element to learn, we have that $\mathcal{E}^p_L \neq \emptyset$ and $\mathcal{E}^q_L = \emptyset$. Let $D_L : \mathcal{A} \to \mathcal{L}$ be an explanation of L according to p.

Cats, by definition

Define the category $\mathcal{F}^q$ adding the limit $\hat{L}^q$ of the diagram

$$\mathcal{A} \xrightarrow{D_L} \mathcal{L} \xrightarrow{\nabla q^\sharp} \mathbf{Set}$$

as a fiber over L.

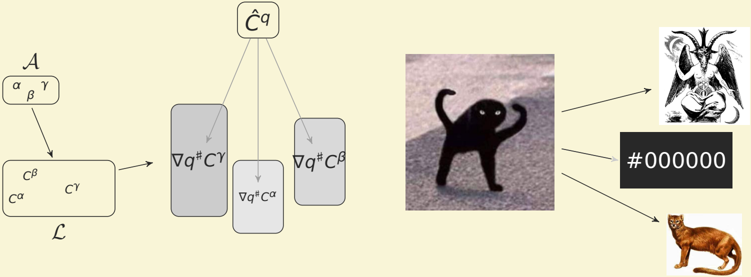
The *new linguistic framework of q* after learning L is the factorization

$$T = \left(\mathcal{F}^q \xrightarrow{s} \mathcal{E}^{\tilde{q}} \xrightarrow{\tilde{q}^\sharp} \mathcal{L}\right).$$

p knows what is L; splits the concept as the intersection (or ‘limit’) of a certain number of simpler, atomic concepts. This is the finite diagram $D_L : \mathcal{A} \to \mathcal{L}$ whose limit is L.

p gives q the pair $(\mathcal{A}, D_L)$ and a cone towards $\lim D_L$; q adds the information coming from this piece of data to their mental image.

Then, computes; q is left with a new concept, $\lim D_L$, obtained as a limit of simpler concepts that they possess, and organically fitting ino their previous image of the world.



The concept of a ‘cat’ can be obtained as the limit of a certain diagram of elements having values in the fibres over concepts like ‘evil’, ‘feline’, and ‘black’.

Future prospects

Describe communication in FibLang.

At the very least, communication can happen in three different ways:

- messaging is a dynamical process;
- translation is an algorithmic process;
- mutual understanding is a game-theoretic process.

Future prospects

Study the double category having typical cell a square

$$\begin{array}{ccc} \mathcal{E} & \relbar\joinrel\mapstochar\joinrel\rightarrow & \mathcal{F} \\ {\scriptstyle p}\big\downarrow & & \big\downarrow{\scriptstyle q} \\ \mathcal{L} & \relbar\joinrel\mapstochar\joinrel\rightarrow & \mathcal{L}' \end{array}$$

with vertical maps fibrations, and horizontal maps profunctors.