Geometry of categories, categories of geometry

August 6-10, @SISSA

Simplicial Sets: a breakthrough in homotopy theory.

MAURO PORTA (Université Paris XI). The theory of simplicial sets has been developed to give combinatorial tools in order to deal with the homotopy of topological spaces. Since its introduction, much work has been done and by now they play a key role in a large part of Mathematics (both in foundational setting and as computational tools). In this series of talks, we will build simplicial sets from the scratch, paying a particular attention to their homotopy theory. The subject being particularly hard for the beginner, we will develop in great detail some technical aspects of the theory, to give a feeling of the standard techniques used by the experts (topics include anodyne extensions and minimal fibrations). We will end by giving an overview of simplicial methods in homological algebra.

REFERENCES.

J. P. May, *Simplicial Objects in Algebraic Topology*.
P. G. Goerss, J. F. Jardine, *Simplicial Homotopy Theory*.
C. Weibel, *Introduction to homological algebra*.

Model categories: an introduction

MAURO PORTA (Université Paris XI). Model categories were introduced by Quillen at the end of '60s, attempting to give an axiomatic treatment to homotopy theory, at the time when the mathematical community began understading that applications were possible even outside the simple topological setting. This series of talks can be splitted into three parts: introduction, advanced topics and applications. In the first one, we will cover the basics of the theory such as the small object argument, the various notions of being cofibrantly generated, cellular, proper, simplicial etc., as well as several standard techniques like the recognition theorem and the lifting criterion. In a second moment, we will discuss localizations for model categories; we will deal with two (really) different kind of localizations: the Dwyer - Kan theory (leading naturally to the notion of mapping space) and the Bousfield localization. We will end explaining how to use model categories to approach the theory of higher stacks.

REFERENCES.

M. Hovey, *Model categories*.
P. Hirschhorn, *Model categories and their localization*.
W.G. Dwyer, D.M. Kan, *Simplicial localization of categories*.
S. Hollander, *A homotopy theory for stacks*.
J.F. Jardine, *Simplicial presheaves*.
B. Toen and G. Vezzosi, *Homotopical algebraic geometry* II.

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"This is the end, my holim friend!"

FOSCO LOREGIAN (SISSA) . Introduced by Yoneda and Kelly, the formalism of (co)ends is able to subsume in a really neat fashion various constructions

- in elementary Category Theory (natural transformations between two functors, Kan extensions, "tensor product" of functors),
- in Abstract Algebra (tensor product of R-modules, induced and coinduced representation of a group along a morphism),
- in Geometry/Topology (geometric realization of a simplicial set, the nerve-realization paradigm, the classifying space of a topological monoid),
- and in less elementary Category Theory (the theory of Benabou's profunctors) and less elementary geometry/topology (characterization and generalization of May's operads, Borsuk-Cordier-Porter's shape theory via profunctors).

My aim is to convey the idea that an "endy" approach to Category Theory can turn involved arguments in neat, one-line-long proofs.

REFERENCES.

S. Mac Lane, *Categories for the Working Mathematician*.
S. Mac Lane, *The milgram bar construction as a tensor product of functors*.
E. Riehl, *Categorical Homotopy Theory*.
J. Bénabou, *Distributors at work*.
J. M. Cordier, T. Porter, *Shape Theory: Categorical methods of approximation*.

An homotopical view on stacks

FOSCO LOREGIAN (SISSA) . In their paper "Strong stacks and classifying spaces" A. Joyal and M. Tierney provide an internal characterization of the classical (or "folk") model structure on the category of groupoids in a Grothendieck topos $\mathscr{E}$. The fibrant objects in the classical model structure on $\mathbf{Gpd}(E)$ are called *strong stacks*, as they appear as a strengthening of the notion of *stack* in $\mathscr{E}$ (i.e. an internal groupoid object subject to a certain *descent* condition). The main application is when $\mathscr{E}$ is the topos of simplicial sheaves on a space or on a site: in that case strong stacks are intimately connected with classifying spaces of simplicial groups.

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Higher topos theory: a quasicategorical approach

MAURO PORTA (Université Paris XI). Taking motivations from the previous series of talks, we develop the foundations of homotopical algebra and derived algebraic geometry following the approach of J. Lurie. We will briefly discuss $(\infty, 1)$-categories in an abstract setting, but we will stick with the language of quasicategories for the most of the time. Several technical tools will be introduced (relative joins, simplicial nerve, inner fibrations, left fibrations etc.); we will explain the relationship between model categories and $(\infty, 1)$-categories, paying a special attention to some preservation theorem (mostly concerning limits and adjunctions). In a second part, we will turn to the basic notions of higher algebra: we will discuss stable $(\infty, 1)$-categories, spectra and, time permitting, E_∞-rings.

REFERENCES.

J. Lurie, *Higher topos theory*.
J. Lurie, *Higher algebra*.

Simplicial presheaves: a homotopical approach to (derived) stacks

MAURO PORTA (Université Paris XI). In their classical article *Irreducibility of the space of curves of given genus*, Deligne and Mumford settled down the basics for the theory of stacks. We will give a quick overview of the classical definition passing through the notion of fibered category and discussing the equivalence between (lax) functors with values in **Grpd**, the category of groupoids. Observing that **Grpd** can be seen as nullification of simplicial sets, we explain how to define a notion of higher (Deligne-Mumford) stack using simplicial presheaves; we will discuss in detail the model category structure discovered by Jardine, as well as several characterization for its fibrations in term of (local) lifting properties. We end by sketching a second further generalization that naturally lead to the notion of higher derived stack in the sense of Toen and Vezzosi (HAG II).

REFERENCES.

S. Hollander, *A homotopy theory for stacks*.
J.F. Jardine, *Simplicial presheaves*.
D. Dugger and I. Isaksen, *Weak equivalences for simplicial presheaves*.
D. Dugger, S. Hollander and I. Isaksen, *Hypercovers and simplicial presheaves*.
B. Toen and G. Vezzosi, *Homotopical algebraic geometry II*.

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Homotopy Type Theory.

ANDREA GAGNA (Università degli studi di Milano). Homotopy type theory is a new branch of mathematics that combines aspects of several different fields in a surprising way. It is based on a recently discovered connection between homotopy theory and type theory. It touches on topics as seemingly distant as the homotopy groups of spheres, the algorithms for type checking, and the definition of weak ∞-groupoids. Homotopy type theory offers a new “univalent” foundation of mathematics, in which a central role is played by Voevodsky’s univalence axiom and higher inductive types. The present book is intended as a first systematic exposition of the basics of univalent foundations, and a collection of examples of this new style of reasoning – but without requiring the reader to know or learn any formal logic, or to use any computer proof assistant. We believe that univalent foundations will eventually become a viable alternative to set theory as the “implicit foundation” for the unformalized mathematics done by most mathematicians.
(intro taken from `http://homotopytypetheory.org/book/`)

A general theory of integral transforms.

FOSCO LOREGIAN (SISSA). To be written…

	06/08/13	07/08/13	08/08/13	09/08/13	10/08/13
10:00 11:00	**Model**	**Sset**	**Model**	**HTT**	**SiPre**
11:00 12:00	**Model**	**Ends**	**HoTT**	**HTT**	**HoSta**
12:00 13:00	**Ends**	**Ends**	**HoTT**	**FuMu**	**HoSta**
14:00 15:00	**HoTT**	**Model**	**Model**	**FuMu**	**SiPre**
15:00 16:00	**Sset**	**Model**	**Model**	**SiPre**	**SiPre**

Model = Model Categories: an introduction
HoTT = Homotopy Type Theory
Ends = Intrduction to ends and coends
FuMu = A general theory of integral transforms
HoSta = Homotopical view on stacks
SiPre = Simplicial Presheaves
HTT = Higher Topos Theory
Sset = Simplicial Sets

COG
GOC

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As for its philosophical side, GOC-COG can be defined as an *experimental window open to autonomous research*, where the word “research” has to be understood in etymological sense: the daily struggle of a bunch of curious minds towards Gnosis, the firm determination to avoid the fragmented, edonistic tendency of a certain modern mathematical practice, which concentrates collective efforts on solving a particular instance of a problem instead of *building a theory* eroding our questions millennium after millennium.

Geometry is, and will always be, the *terra incognita* where new mathematical structures can be found, new phenomena observed, new/old ideas stretched to solve old/new problems; after more than half-a-century of evolution, it shouldn’t be a surprise that Category Theory is becoming the natural language to do Geometry, or better to say, *to constantly expand the meaning of the word “Geometry”*. The other side of the coin is that categories are really *geometric objects*: their nature is best understood when geometric ideas (model categories, simplicial stuff, topological and homotopical techniques. . .) come into play. This is the reason for the intertwining of COG (category theory adapted to understand geometry) and GOC (geometric ideas adapted to understand what categories really are).

A couple of words about the *technical* side of our experiment.

Interventions will be computed in *talks* (which id our fundamental unit of measurement): a single *talk* denotes an amount of time between 45 and 60 minutes. A tentative timetable should contain 5 talk per day, each interspersed with a break of (at least) 15 minutes for discussions and other stuff (usually chatting about Mathematics or smoking a cigarette; nevertheless we are open to other suggestions, e.g. –being the August in TS quite hot– some applications of the “share your own beer” principle).

However, keep in mind that we do not want to be particularly strict about this point: *the MATHer the better* (or in other words: go on ’til your chalk is finished, and if you need more time then carve the blackboard with your nails).

Again, keep in mind that you will not find perfectly polished ideas; we keep learning, as everyone does, hence we keep making mistakes, as everyone does. We definitely prefer real Mathematics to boring lectures one repeated the evening before his talk: questions, fragile ideas, conjectures, unexpected links between different expositions, oniric intuitions allowing a rapid lurk to the Catalogue of all catalogues are not only allowed, but warmly encouraged.

COG
GOC

A.G.
M.P.
F.L.