Enriching Yoneda: A gentle introduction to Enriched Category Theory.

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Abstract

Enriched Category Theory provides a systematic setting to formalise the intuitive idea of dealing with "categories" where the hom-sets are actually objects in (another) category $\mathscr{V}$. Technically speaking, one realises this intention by considering a class of objects, $\mathrm{Ob}(\mathscr{A})$, and requiring that, for each $A, B \in \mathrm{Ob}(\mathscr{A})$, it is endowed with an object $\mathscr{A}(A, B)$ belonging to a (fixed) monoidal category $\mathscr{V}$. The existence of suitable composition laws and unities is then added to the axioms defining an enriched category, so as to give a perfect parallelism with the usual notion of a category. Once the right initial definition is properly given, all the other basic concepts of ordinary Category Theory can be (quite) easily enriched.
In particular, having enriched the notions of functor and natural transformation, one might think that also an enriched version of a fundamental theorem such as the Yoneda Lemma follows readily. Unfortunately, this is not completely true. Indeed, the very statement of this result is inspired by the quite deep and elegant characterization of the class of (ordinary) natural transformations between two (ordinary) functors as a suitable end.

In this talk, we will formulate and prove the aforementioned enriched Yoneda Lemma, introducing (hopefully) all the needed notions to understand it, hence presuming no preliminary knowledge of Enriched Category Theory in the audience. Concerning prerequisites, a basic familiarity with the most elementary concepts and definitions of Category Theory is assumed. The notions and initial results regarding monoidal categories will also be needed, but they might (and probably will) be recalled. Any sort of knowledge of dinatural transformations and ends may prove useful, but for sure they will not be strictly necessary to follow the lecture.

The talk will be given in English, unless there are no foreign students and the audience members prefer the Italian language.

References

[1] Francis Borceaux, *Handbook of Categorical Algebra 1, Basic Category Theory*, Encyclopedia of Mathematics and its Applications 50, Cambridge University Press, Cambridge, 1994.

[2] Francis Borceaux, *Handbook of Categorical Algebra 2, Categories and Structures*, Encyclopedia of Mathematics and its Applications 50, Cambridge University Press, Cambridge, 1994.

[3] G. M. Kelly, *Basic Concepts of Enriched Category Theory*, London Mathematical Society Lecture Note Series, 64, Cambridge University Press, Cambridge, 1982.

[4] Saunders Mac Lane, *Categories for the Working Mat1hematician*, Second Edition, Graduate Texts in Mathematics, Springer-Verlag, New York, 1997.